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The Production Routing Problem: A Review of Formulations and Solution Algorithms

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Abstract. The production routing problem (PRP) combines the lot-sizing problem and the vehicle routing problem, two classical problems that have been extensively studied for more than half a century. The PRP is solved in an attempt to jointly optimize production, inventory, distribution and routing decisions and is thus a generalization of the inventory routing problem (IRP). Although the PRP has a complicated structure, there has been a growing interest in this problem during the past decade in both academia and industry. This article provides a comprehensive review of various solution techniques that have been proposed to solve the PRP. We attempt to provide an in-depth summary and discussion of different formulation schemes and of algorithmic and computational issues. Finally, we point out interesting research directions for further developments in production routing.

Keywords. Integrated supply chain planning, production routing, inventory routing, exact algorithms, heuristics, review.

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1. Introduction

In a typical supply chain which consists of sequential activities of production, storage and distribution, each individual process is often planned and optimized using predetermined decisions from its preceding activities. For example, a production planner makes production lot-sizing decisions in order to minimize production and inventory costs at the production facility. The planned lot-sizing decisions are then used as inputs in subsequent steps of distribution planning. Since the decisions are limited by the plan of the former process, the benefits of coordination in the planning process have been left behind. An integrated supply chain operational planning system is a tool that is used to jointly optimize several planning decisions thereby capturing the additional benefits of coordination between sequential activities in the chain. In recent years, many companies, such as Kellogg (Brown et al., 2001) and Frito-Lay (Çetinkaya et al., 2009), have set up integrated planning systems and achieved multi-million cost savings. The key to success is an application that is not only able to produce solutions with minimal costs, but that can also be used in an effective and timely manner.

The production routing problem (PRP) is an integrated operational planning application that jointly optimizes production, inventory, distribution and routing decisions. It is of practical relevance in a Vendor Managed Inventory (VMI) approach, in which the supplier monitors the inventory at retailers and also decides on the replenishment policy for each retailer. The supplier acts as the central decision maker who solves an integrated supply chain planning problem. The advantage of a VMI policy with respect to the traditional retailer managed inventory lies in a more overall efficient resource utilization. The PRP connects two well-known problems, namely the lot-sizing problem (LSP) and the vehicle routing problem (VRP), to produce an optimal solution when considering the total system cost. The PRP is also a generalization of the lot-sizing problem with direct shipment and of the inventory routing problem (IRP). Solving the PRP becomes challenging as it is a combined version of the LSP and VRP and it incorporates the constraints of these two difficult problems. We aim to provide an in-depth review of the PRP, particularly with respect to the formulations and solution algorithms. Different formulation schemes of the PRP are examined. The approaches to compute lower bounds, exact algorithms and heuristics are thoroughly reviewed. We further discuss future research directions.

In the rest of this section, we first provide a brief overview of the three

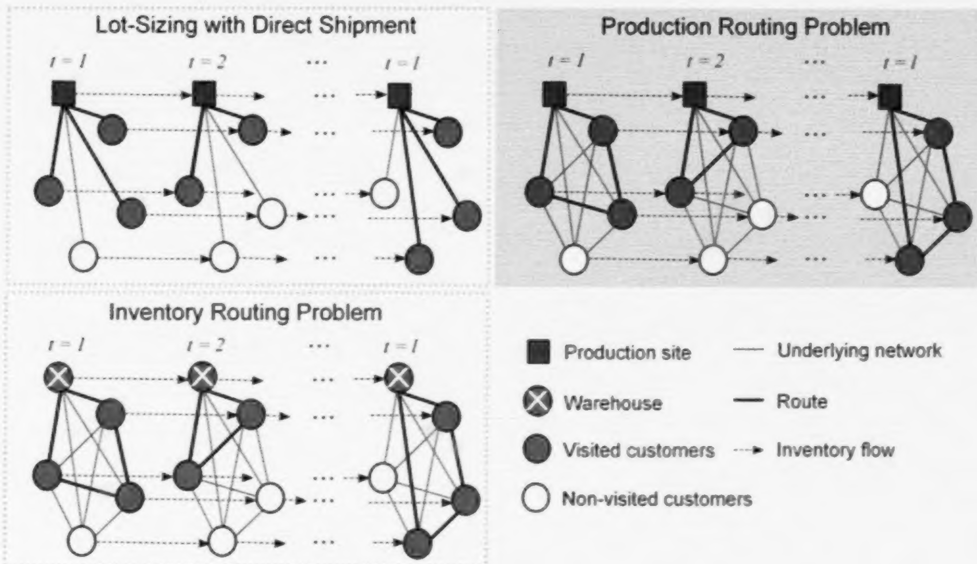


Figure 1: Network Representations of the Integrated Problems

integrated problems. The network representations of these problems in the case of a single supplying facility and multiple customers in a discrete time finite horizon are presented in Figure 1. Note that the supplying facility can be a *plant* with setup costs and production decisions or a *warehouse* with fixed ordering costs and ordering decisions. In each period, a single or multiple products can be made available at the supplying facility and they are transported to the customers in order to satisfy demands. The products can be stored at the plant or at the customers, thus incurring inventory holding costs.

1.1. Integrated Lot-Sizing with Direct Shipment

In this problem, the products are directly transported from the manufacturing plant to the customers. The production, setup, inventory and direct shipment costs are minimized over the planning horizon. This problem typically incorporates various production aspects, e.g., production setup cost and/or setup time, and involves distribution decisions where the fixed and unit costs of delivery are customer specific.

The integrated production and direct shipment distribution planning was studied by several researchers. Most of the studies considered the distribu-

tion cost as a fixed cost or a complex cost function. Li et al. (2004) focused on the lot-sizing problem with a piecewise linear transportation cost function where the supplier has the option to deliver by direct delivery with truckload (TL) or less-than-truckload (LTL) transportation. They developed a dynamic programming approach to solve the one-product and one-customer problem. Jaruphongsa et al. (2007) proposed other dynamic programming algorithms to solve the problem with TL and LTL cost structures. A problem with a special cost structure, where the supplier can get a discount from transportation capacity reservation, was studied by van Norden and van de Velde (2005). A more general piece-wise linear transportation cost function was addressed by Rizk et al. (2006). They decomposed the integrated problem into uncapacitated lot-sizing and time-independent subproblems and applied a Lagrangian relaxation technique to obtain lower bounds. In the more general case of multiple customers, Chand et al. (2007) developed a dynamic programming algorithm to solve the problem in which backlogging is allowed. Jaruphongsa and Lee (2008) considered the problem with split delivery under time window restrictions and employed dynamic programming algorithms to solve the problem. A special problem of lot-sizing with truckload shipment where transshipments between the customers are allowed was considered by Herer and Tzur (2001). The multi-item problem with one customer was considered by Lee et al. (2005). In the case of uncapacitated production and uncapacitated vehicles, the problem with direct shipments is also known as the one-warehouse multi-retailer problem (OWMR). Federgruen and Tzur (1999) considered the OWMR with multiple items and developed a time-partitioning heuristic to solve the problem. Solyalı and Süral (2012) proposed a new strong formulation based on the combined transportation and shortest path model to solve the OWMR with a single product. Melo and Wolsey (2012) discussed several formulations and proposed hybrid heuristics for the two-level production-transportation problems with capacitated production and vehicles.

There is a link between the lot-sizing problem with truckload cost structure and the classical lot-sizing problem with batch size where the batch quantity is smaller than the maximum production quantity in one period. The truck capacity can be viewed as the fixed batch quantity limit and the cost of dispatching one truck can also be considered as the fixed production cost of one batch. There is also a link between the lot-sizing problem with transshipments and the lot-sizing problem with production substitution where a product can be used to substitute for the demand of another product

(Hsu et al., 2005). The cost of transshipment between customer locations can be viewed as the cost of production substitution.

1.2. Inventory Routing Problem (IRP)

When the routing aspect is included and the production aspect is disregarded, the problem is transformed into the inventory routing problem (IRP). In the IRP, the starting point is a warehouse where there is no production decision as the production quantities made available in each period are typically given. A vehicle can visit more than one customer by travelling along its route. As a generalization of the VRP, which consists of the decisions on delivery quantities and routes to serve customers, the IRP also includes the timing to serve the customers' demands. This makes the problem much more difficult than the classical VRP due to the complex periodic routing and inventory decisions. The IRP is obviously NP-hard since it contains the VRP as a special case (Coelho et al., 2012b).

The IRP first appeared in a gas delivery study by Bell et al. (1983). The problem was solved using a Lagrangian relaxation method and was decomposed by time period and by vehicle. Carter et al. (1996) and Campbell and Savelsbergh (2004) proposed efficient heuristic procedures by decomposing the IRP into an allocation problem (AP) and a vehicle routing problem (VRP). Since the IRP is a complicated combinatorial problem, several metaheuristics, e.g., tabu search (Rusdiansyah and Tsao, 2005), genetic algorithm (Abdelmaguid and Dessouky, 2006), greedy randomized adaptive search procedure (GRASP) (Savelsbergh and Song, 2007), hybrid heuristic with combined tabu search and MIPs (Archetti et al., 2011), and adaptive large neighborhood search (ALNS) (Coelho et al., 2012a,b), have been proposed. Gaur and Fisher (2004) discussed a periodic IRP where the demand pattern is repeated and developed a heuristic to solve the problem.

As mentioned in Andersson et al. (2010), few exact algorithms have been proposed to solve the IRP due to its complexity. Notable exceptions include a branch-and-cut procedures to solve the IRP with a single capacitated vehicle by Archetti et al. (2007) and Solyalı and Süral (2011). Archetti et al. (2007) introduced several valid inequalities to solve the problem under three different inventory replenishment policies. In the first policy, called order-up-to level (OU), a visited customer receives exactly the amount which brings its inventory up to a predefined target stock level (TSL). The second policy, called maximum level (ML), allows delivery quantities to be any positive value but the inventory at each customer cannot exceed its maximum stock

level. The third policy is similar to the ML policy but there is no maximum stock level imposed at the customers. Solyalı and Süral (2011) strengthened the formulation of the IRP with the OU policy of Archetti et al. (2007) by using a shortest path network reformulation. Adulyasak et al. (2013) and Coelho and Laporte (2013) extended the approach of Archetti et al. (2007) to the IRP with multiple vehicles. The IRP with transshipment and the IRP with several consistency features were also considered by Coelho et al. (2012a).

Variants of the IRP have been proposed as well. Christiansen (1999) introduced an IRP application in a maritime context, called the inventory pickup and delivery problem, and applied a Dantzig-Wolfe decomposition and column generation approach to solve the problem. Savelsbergh and Song (2008) considered the IRP with continuous moves where a product is distributed from a set of plants to a set of customers by multiple vehicles. The authors developed a branch-and-cut approach to solve the problem. We refer to Andersson et al. (2010) and Coelho et al. (2013) for more comprehensive reviews of the IRP.

1.3. Production Routing Problem (PRP)

The two integrated problems discussed in the previous sections each disregard one important aspect of the supply chain operational planning process, i.e., the integrated lot-sizing problem with direct shipment does not incorporate routing decisions, while the IRP disregards the production part. In the PRP, the plant must decide in each period whether or not to make the product and determine the corresponding lot size. If production does take place, this process incurs a fixed setup cost as well as unit production costs. In addition, the lot size cannot exceed the production capacity. Deliveries are made from the plant to the retailers by a limited number of capacitated vehicles and routing costs are incurred. If products are stored at the plant or at the retailers, unit inventory holding costs are also incurred. Table 1 provides a summary of the PRP literature.

The PRP has received more attention in recent years. The benefits of coordination in the PRP were first discussed by Chandra (1993) and Chandra and Fisher (1994). They showed that 3-20% cost savings can be achieved by solving the PRP compared to sequentially solving the separate problems. As for the case of the IRP, most of the previous studies employed heuristic procedures to solve the problem. Several metaheuristics, such as GRASP (Boudia et al., 2007), memetic algorithm (Boudia and Prins, 2009), tabu search (Bard

and Nananukul, 2009b; Armentano et al., 2011), and ALNS (Adulyasak et al., 2012b) have been employed. Archetti et al. (2011) discussed the PRP under the ML and OU policies and developed a mixed integer programming (MIP) heuristic to solve the problem. Bard and Nananukul (2009a, 2010) introduced a heuristic based on a branch-and-price framework.

Due to the complexity of the problem, few studies have introduced exact algorithms or methods to compute strong lower bounds. Fumero and Vercellis (1999) and Solyalı and Süral (2009) developed a Lagrangian relaxation approach to obtain lower bounds based on the multi-commodity flow formulation. Ruokokoski et al. (2010) and Archetti et al. (2011) employed a branch-and-cut approach similar to that of Archetti et al. (2007) to solve the PRP. Ruokokoski et al. (2010) explored different lot-sizing reformulations for the PRP with uncapacitated production and a single uncapacitated vehicle, while Archetti et al. (2011) focused on the PRP with uncapacitated production and a single capacitated vehicle, and introduced several valid inequalities to solve the problem. Adulyasak et al. (2013) focused on the PRP with multiple vehicles and proposed two branch-and-cut approaches based on different formulations schemes to solve the problem.

Table 1: Summary of algorithms for the PRP

Author(s)	Problem Characteristics									
	Production			Inventory		Distribution			Solution Method	
	N.Plants	Product	Cap.	Policy	Cap.	Fleet	N.Vehs	Cap.	Type	Approach
Chandra (1993)	Single	Multiple	✓	ML		Hom.	Unlimited	✓	H	Decomposition
Chandra and Fisher (1994)	Single	Multiple	✓	ML		Hom.	Unlimited	✓	H	Decomposition
Fumero and Vercellis (1999)†	Single	Multiple	✓	ML		Hom.	Limited	✓	H/L	Lagrangian Relaxation
Lei et al. (2006)	Multiple	Single	✓	ML	✓	Het.	Limited	✓	H	Decomposition
Boudia et al. (2007)	Single	Single	✓	ML	✓	Hom.	Limited	✓	H	GRASP
Boudia et al. (2008)	Single	Single	✓	ML	✓	Hom.	Limited	✓	H	Decomposition
Boudia and Prins (2009)	Single	Single	✓	ML	✓	Hom.	Limited	✓	H	Memetic
Bard and Nananukul (2009b)	Single	Single	✓	ML	✓	Hom.	Limited	✓	H	Tabu search
Solyalı and Süral (2009)	Single	Single		OU	✓	Hom.	Limited	✓	H/L	Lagrangian Relaxation
Bard and Nananukul (2009a, 2010)	Single	Single	✓	ML	✓	Hom.	Limited	✓	H/L	Branch-and-price
Ruokokoski et al. (2010)	Single	Single		ML		Hom.	Single		E	Branch-and-cut
Armentano et al. (2011)	Single	Multiple	✓	ML	✓	Hom.	Limited	✓	H	Tabu search
Archetti et al. (2011)	Single	Single		ML/OU	✓	Hom.	Single	✓	E/H	Branch-and-cut/MIP Heuristic
Adulyasak et al. (2012b)	Single	Single	✓	ML	✓	Hom.	Multiple	✓	H	ALNS
Adulyasak et al. (2013)	Single	Single	✓	ML/OU	✓	Hom.	Multiple	✓	E/H	Branch-and-cut/ALNS
Adulyasak et al. (2012a)‡	Single	Single	✓	ML/OU	✓	Hom.	Multiple	✓	E/H	Branch-and-cut/ALNS
Absi et al. (2013)	Single	Single		ML	✓	Hom.	Multiple	✓	H	Iterative MIP Heuristic

Note. Hom. - Homogeneous, Het. - Heterogeneous; H - Heuristic, E - Exact, L - Approach to compute lower bound

†considered a PRP variant with unit transportation costs

‡considered a two-stage stochastic PRP with demand uncertainty

The paper is organized as follows. Section 2 provides the problem description and formulations of the basic version of the PRP which consists of a network of a single product, a single plant and multiple customers. Solution approaches for several variants of the PRP are discussed next. Section 3 provides details on the approaches developed to compute lower bounds and Section 4 reviews recent developments in exact algorithms. Section 5 presents various heuristics and recent computational results. We further discuss future research opportunities in Section 6 and this is followed by the conclusion.

2. Notation and Formulations for the PRP

2.1. Description and Notation of the PRP

A PRP network is defined on a complete directed graph $G = (N, A)$ where N represents the set of the plant and the customers indexed by $i \in \{0, ..., n\}$ and $A = \{(i, j) : i, j \in N, i \neq j\}$ is the set of arcs. The plant is represented by node 0 and we further define the set of customers $N_c = N \setminus \{0\}$. Over a finite set of time periods $T = \{1, \ldots, l\}$, a single product can be produced at the plant and delivered by a set of identical vehicles $K = \{1, \ldots, m\}$ to the customers to satisfy the demands in each period. The parameters and the decision variables are defined as follows.

Parameters:

u unit production cost;

f fixed production setup cost;

h_i unit inventory holding cost at node i;

c_{ij} transportation cost from node i to node j;

d_{it} demand at customer i in period t;

C production capacity;

Q vehicle capacity;

L_i maximum or target inventory level at node i;

I_{i0} initial inventory available at node i.

Decision variables:

p_t production quantity in period t;

I_{it} inventory at node i at the end of period t;

y_t equal to 1 if there is production at the plant in period t, 0 otherwise;

z_{0t} the number of vehicles leaving the plant in period t;

z_{it} equal to 1 if customer i is visited in period t, 0 otherwise, $\forall i \in N_c$;

x_{ijt} if a vehicle travels directly from node i to node j in period t, 0 otherwise;

q_{it} quantity delivered to customer i in period t;

w_{it} load of a vehicle before making a delivery to customer i in period t.

We further let $M_t = \min\left\{C, \sum_{j=t}^{l} \sum_{i \in N_c} d_{ij}\right\}$ and $\widetilde{M}_{it} = \min\left\{L_i, Q, \sum_{j=t}^{l} d_{ij}\right\}$.

2.2. Formulations for the PRP

We first present a model based on the basic LSP and VRP formulations. It is also the most compact one as it contains a polynomial number of constraints. The PRP is formulated with variables that control the amounts delivered by a homogenous fleet of vehicles. A basic formulation based on that of Bard and Nananukul (2009a, 2010) is as follows.

$$(PRP1): \min \sum_{t \in T} \left(up_t + fy_t + \sum_{i \in N} h_i I_{it} + \sum_{(i,j) \in A} c_{ij} x_{ijt} \right) \tag{1}$$

$$
\begin{aligned}
\text{s.t.} \quad & I_{0,t-1} + p_t = \sum_{i \in N_c} q_{it} + I_{0t} && \forall t \in T && (2)\\
& I_{i,t-1} + q_{it} = d_{it} + I_{it} && \forall i \in N_c, \forall t \in T && (3)\\
& p_t \leq M_t y_t && \forall t \in T && (4)\\
& I_{0t} \leq L_0 && \forall t \in T && (5)\\
& I_{i,t-1} + q_{it} \leq L_i && \forall i \in N_c, \forall t \in T && (6)\\
& q_{it} \leq \widetilde{M}_{it} z_{it} && \forall i \in N_c, \forall t \in T && (7)\\
& \sum_{j \in N} x_{ijt} = z_{it} && \forall i \in N_c, \forall t \in T && (8)\\
& \sum_{j \in N} x_{jit} + \sum_{j \in N} x_{ijt} = 2 z_{it} && \forall i \in N, \forall t \in T && (9)\\
& z_{0t} \leq m && \forall t \in T && (10)\\
& w_{it} - w_{jt} \geq q_{it} - \widetilde{M}_{it}(1 - x_{ijt}) && \forall (i,j) \in A, \forall t \in T && (11)\\
& 0 \leq w_{it} \leq Q z_{it} && \forall i \in N_c, \forall t \in T && (12)\\
& p_t, I_{it}, q_{it} \geq 0 && \forall i \in N, \forall t \in T && (13)\\
& y_t, x_{ijt} \in \{0,1\} && \forall i, j \in N, \forall t \in T && (14)\\
& z_{it} \in \{0,1\} && \forall i \in N_c, \forall t \in T && (15)\\
& z_{0t} \in \mathbb{Z}^+ && \forall t \in T. && (16)
\end{aligned}
$$

The objective function (1) minimizes the total production, setup, inventory and routing costs. Constraints (2)-(6) represent the lot-sizing part of the problem. Constraints (2) and (3) are the inventory flow balance at the plant and customers, respectively. Constraints (4) are the setup forcing and production capacity constraints. The constraints force the setup variable to be one if production takes place in a given period and limit the production quantity to the minimum value between the production capacity and the total demand in the remaining periods. Constraints (5) and (6) limit the maximum inventory at the plant and customers, respectively. The inventory part of this model is controlled by the so-called maximum level (ML) policy as defined by Archetti et al. (2007), where the delivery quantity can be any positive number but the resulting inventory level after delivery prior to demand consumption cannot exceed the maximum inventory level. The remaining constraints, i.e., (7)-(12), are the vehicle loading and routing restrictions. Constraints (7) allow a positive delivery quantity only if customer

i is visited in period t and each customer can be visited by at most one vehicle (8). Constraints (9) are the vehicle flow conservation. Constraints (10) limit the number of trucks that can be used to the number of available trucks. Constraints (11) are the vehicle loading restrictions and subtour elimination constraints in the form of the Miller-Tucker-Zemlin inequalities (Miller et al., 1960). These constraints do not allow taking an arc that generates a subtour as shown in Figure 2; the arc $(3, 1, t)$ cannot be taken because $w_{1t} - w_{2t} \geq q_{1t}$ is not valid. Constraints (12) are the vehicle capacity constraints.

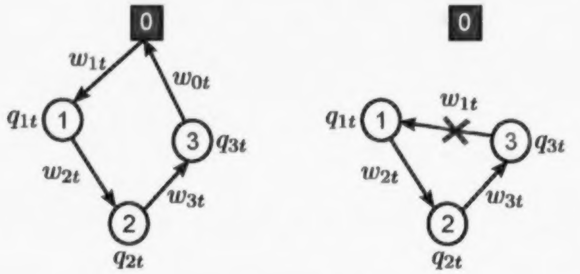


Figure 2: Illustration of vehicle restriction and subtour elimination constraints (11).

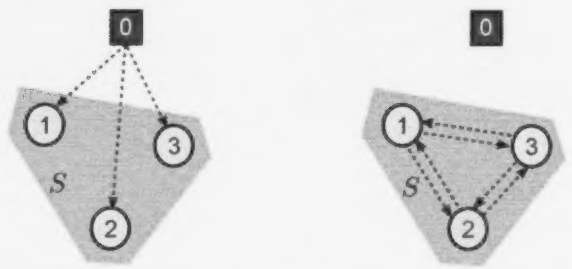
In Bard and Nananukul (2009a, 2010), the subtour elimination constraints (11) and vehicle capacity (12) constraints are used. However, in the VRP, this subtour elimination constraint set can lead to a weak formulation in the routing part (Toth and Vigo, 2001). Constraints (11) and (12) can be replaced with other subtour elimination constraints, i.e.,

- fractional capacity constraints (FCCs) (Letchford and Salazar-González, 2006) as presented in Chandra and Fisher (1994):

$$\sum_{i \notin S} \sum_{j \in S} x_{ijt} \geq \sum_{i \in S} q_{it}/Q \quad \forall S \subseteq N_c : |S| \geq 1, \forall t \in T \tag{17}$$

- generalized fractional subtour elimination constraints (GFSECs):

$$\sum_{i \in S} \sum_{j \in S} x_{ijt} \leq |S| - \sum_{i \in S} q_{it}/Q \quad \forall S \subseteq N_c : |S| \geq 2, \forall t \in T. \tag{18}$$



$x_{01t} + x_{02t} + x_{03t} \geq 1$ $\qquad$ $x_{12t} + x_{21t} + x_{13t} + x_{31t} + x_{23t} + x_{32t} \leq 2$

Figure 3: Illustration of FCCs (left) and GFSECs (right)

Figure 3 depicts how these constraints eliminate subtours. Suppose that $(q_{1t} + q_{2t} + q_{3t})/Q = 1$. The FCC forces one arc from outside the set S to connect to a node inside S, while the GFSEC does not allow more than two arcs in the set S. Both of the constraints can eliminate subtours in the set S.

Unlike in the VRP where the delivery quantity to each customer is known a priori, the value $(\sum_{i \in S} q_{it})/Q$ cannot be rounded up in the PRP because the delivery quantity q_{it} is a decision variable and this would result in a non-linear formulation. Adulyasak et al. (2013) used a strengthened version of constraints (18) where the parameter $|S|$ is replaced with $\sum_{i \in S} z_{it}$. They also observed that the form (18) is numerically unstable due to the fractional term q_{it}/Q. The alternative form of the GFSECs presented in Adulyasak et al. (2013) is as follows:

$$Q \sum_{i \in S} \sum_{j \in S} x_{ijt} \leq \sum_{i \in S} (Q z_{it} - q_{it}) \quad \forall S \subseteq N_c : |S| \geq 2, \forall t \in T. \tag{19}$$

However, using these constraints in the formulation instead of (11), the problem becomes much larger due to an exponential number of subsets. A branch-and-cut (BC) procedure is typically an efficient approach to solve the problem. In this procedure, these constraints are initially removed and added iteratively during the branch-and-bound process. Note that in the case where an undirected graph is assumed, the formulations can be converted by using the method presented by Toth and Vigo (2001).

To overcome the disadvantage of constraints (19) which are fractional, one

can explicitly formulate the problem with a vehicle index and impose routing constraints on each vehicle separately. In this formulation, the variables q_{ikt}, z_{ikt} and x_{ijkt} have the same interpretation as q_{it}, z_{it} and x_{ijt} but they are associated with vehicle k only. The formulation with a vehicle index based on the formulation presented by of Boudia et al. (2007, 2008) is as follows.

$$(PRP2) : \min \sum_{t \in T} \left(up_t + fy_t + \sum_{i \in N} h_i I_{it} + \sum_{(i,j) \in A} c_{ij} \sum_{k \in K} x_{ijkt} \right) \tag{20}$$

s.t.

$$I_{0,t-1} + p_t = \sum_{i \in N_c} \sum_{k \in K} q_{ikt} + I_{0t} \quad \forall t \in T \tag{21}$$

$$I_{i,t-1} + \sum_{k \in K} q_{ikt} = d_{it} + I_{it} \quad \forall i \in N_c, \forall t \in T \tag{22}$$

$$p_t \leq M_t y_t \quad \forall t \in T \tag{23}$$

$$I_{0t} \leq L_0 \quad \forall t \in T \tag{24}$$

$$I_{i,t-1} + \sum_{k \in K} q_{kit} \leq L_i \quad \forall i \in N_c, \forall t \in T \tag{25}$$

$$q_{ikt} \leq \widetilde{M}_{it} z_{ikt} \quad \forall k \in K, \forall i \in N_c, \forall t \in T \tag{26}$$

$$\sum_{k \in K} z_{ikt} \leq 1 \quad \forall i \in N_c, \forall t \in T \tag{27}$$

$$\sum_{j \in N} x_{jikt} + \sum_{j \in N} x_{ijkt} = 2z_{ikt} \quad \forall k \in K, \forall i \in N, \forall t \in T \tag{28}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijkt} \leq |S| - 1 \quad \forall S \subseteq N_c : |S| \geq 2, \forall k \in K, \forall t \in T \tag{29}$$

$$\sum_{i \in N_c} q_{ikt} \leq Q z_{0kt} \quad \forall k \in K, \forall t \in T \tag{30}$$

$$p_t, I_{it}, q_{ikt} \geq 0 \quad \forall i \in N, \forall k \in K, \forall t \in T \tag{31}$$

$$y_t, z_{ikt}, x_{ijkt} \in \{0, 1\} \quad \forall i, j \in N, \forall k \in K, \forall t \in T. \tag{32}$$

The objective function (20) and constraints (21)-(28) are equivalent to the objective function (1) and constraints (2)-(9) in the PRP1, respectively. Con-

straints (29) are in fact the subtour elimination constraints (SECs) similar to those of the travelling salesman problem (TSP) and constraints (30) impose the vehicle capacity on each vehicle. Archetti et al. (2007) and Adulyasak et al. (2013) found that the following form of the SECs, which was originally developed for the selective TSP (Gendreau et al., 1998), is more efficient than (29) when solving the problem with a branch-and-cut algorithm:

$$\sum_{i \in S} \sum_{j \in S} x_{ijkt} \leq \sum_{i \in S} z_{ikt} - z_{ekt} \; \forall S \subseteq N_c : |S| \geq 2, \forall e \in S, \forall k \in K, \forall t \in T. \tag{33}$$

Another advantage of this formulation is that it can be used to solve the problem with a heterogeneous fleet of vehicles, e.g., various fleet size, consumption or costs. In that case, the transportation costs c_{ij} and the vehicle capacity Q in the formulation PRP2 can be replaced with the cost and capacity associated with each vehicle size k, i.e., c_{ijk} and Q_k, respectively.

2.3. Remarks on LSP and VRP Formulation Schemes

Because the PRP includes the structure of the LSP and VRP, this section provides a brief summary of the different formulation schemes for these two problems. For the LSP, a review of basic formulation and reformulation schemes was presented by Pochet and Wolsey (2006). In short, the basic LSP formulation is a weak formulation that gives poor quality lower bounds. Many reformulation schemes can be used to strengthen the formulation. The major LSP reformulation schemes considered for the PRP include the shortest path (Eppen and Martin, 1987) and facility location (Krarup and Bilde, 1977) reformulations. In the single level uncapacitated LSP, these two formulations have the integrality property, i.e., feasible mixed-integer solutions are obtained by solving the LP relaxation. The efficiency of different LSP reformulations in the PRP was studied by Ruokokoski et al. (2010).

In the VRP, different formulations are typically used to solve the problems with different characteristics. For example, the basic formulation is a compact formulation that is suitable for a homogeneous fleet and a limited number of vehicles. To handle a heterogeneous fleet (e.g., various fleet size, consumption, speed, or costs), one should employ a formulation in which a vehicle index can be incorporated, e.g., a multi-commodity flow formulation. In some VRP applications in which the vehicle routes are predefined or have few possibilities, e.g., a truck can only serve the customers in the

same cluster, or in maritime applications where the vessel schedule option is predefined, it is more appropriate to use a set-partitioning formulation. This formulation is also generally used as the master problem in a column generation process. Another issue in VRP formulations is the set of subtour elimination constraints. Applying different subtour elimination constraints can result in different lower bounds (Letchford and Salazar-González, 2006). As in the VRP, some formulations were used in the PRP to deal with specific issues, e.g., a formulation with a vehicle index for the PRP with heterogeneous fleet by Lei et al. (2006) and a path-based formulation used in the column generation approach of Bard and Nananukul (2009a, 2010).

3. Approaches to Compute Lower bounds

As the PRP is a complicated combinatorial optimization problem containing a large number of binary variables, the quality of the lower bound of the basic PRP formulation obtained by solving the LP relaxation is generally very poor. Nananukul (2008) found that the LP relaxation of the formulation PRP1 is not practical in providing relaxed solutions in exact algorithms such as branch-and-bound or measuring the quality of other solution approaches. Hence, alternative relaxation methods have been developed to obtain better lower bounds.

3.1. Lagrangian Relaxation

Lagrangian relaxation (see Fisher (1981)) is an approach to obtain lower bounds by dualizing constraints with Lagrangian multipliers and decomposing the problem into subproblems which are more easily solvable. A Lagrangian relaxation for a variant of the PRP, where unit transportation costs are assumed, was proposed by Fumero and Vercellis (1999). They decomposed the basic LSP and the multi-commodity flow VRP reformulation into subproblems by dualizing the plant inventory constraints and the vehicle capacity constraints. The problem is then transformed into four subproblems, i.e., production (PROD), inventory (INV), distribution (DIS), and routing (ROU) subproblems. The first two subproblems can be solved by inspection and the DIS subproblem can be solved by an LP solver. The lower bound of the ROU subproblem is calculated by the minimum cost network flow problem. Instances with up to 8 periods, 12 customers and 10 products were tested and the algorithm could obtain lower bounds with an average gap of 5.5% compared to an upper bound obtained by a heuristic.

A similar Lagrangian relaxation approach was used by Solyalı and Süral (2009) to solve the PRP with the order-up-to level (OU) policy. However, the lower bounds obtained by this approach were weak compared to the case where the unit transportation costs are used as in Fumero and Vercellis (1999). On the instances with 8 customers and 5 periods, the lower bound produced by the Lagrangian relaxation has an average deviation of 33.16% from the optimal value.

3.2. Column Generation

In a column generation procedure, a basic formulation is decomposed into a restricted master problem (RMP) and subproblems. The original variables are replaced with a convex combination of extreme points of the subproblems which are generated and added iteratively by solving the subproblems. More details about recent general column generation approaches can be found in Lübbecke and Desrosiers (2005).

Bard and Nananukul (2010) proposed a RMP and subproblem formulations for the PRP and developed a branch-and-price procedure. Let $R(t)$ be the sets of delivery plans in period t where a delivery plan, indexed by r, is characterized by the delivery quantity to each customer and routing decisions. The binary variable θ_{rt} is equal to one if the delivery plan r for period t is selected. The parameter c_{rt} is the total cost of using delivery plan r in period t, and μ^i_{rt} is the amount delivered to customer i with delivery plan r in period t. The RMP is formulated as follows.

$$\min \sum_{t\in T}\left(up_t + fy_t + \sum_{i\in N} h_i I_{it} + \sum_{r\in R(t)} c_{rt}\theta_{rt}\right) \quad (34)$$

s.t. (4)-(6), (13) and

$$I_{0,t-1} + p_t = \sum_{i\in N_c}\sum_{r\in R(t)} \mu^i_{rt}\theta_{rt} + I_{0t} \quad \forall t \in T \quad (35)$$

$$I_{i,t-1} + \sum_{r\in R(t)} \mu^i_{rt}\theta_{rt} = d_{it} + I_{it} \quad \forall i \in N_c, \forall t \in T \quad (36)$$

$$\sum_{r\in R(t)} \theta_{rt} \le 1 \quad \forall t \in T \quad (37)$$

$$\theta_{rt} \in \{0,1\} \quad \forall t \in T, \forall r \in R(t). \quad (38)$$

The objective function (34) and constraints (35)-(36) are equivalent to (1) and (2)-(3), respectively. At most one delivery plan can be selected in each period (37).

The subproblem is the delivery schedule generator. Denote by $\alpha_t^1, \alpha_{it}^2$ and α_t^3 the dual variables of the RMP associated with the constraints (35), (36) and (37), respectively. The subproblem is decomposed into a VRP subproblem in each time period. The subproblem for time period t, referred to as $SVRP_t$, is as follows.

$$\min \sum_{(i,j)\in A} c_{ij} x_{ijt} - \sum_{i \in N_c} (\alpha_t^1 + \alpha_{it}^2) q_{it} + \alpha_t^3 \tag{39}$$

s.t. (13)-(16) and

$$q_{it} \leq \widetilde{M}_{it} z_{it} \qquad \forall i \in N_c \tag{40}$$

$$\sum_{j \in N} x_{ijt} = z_{it} \qquad \forall i \in N_c \tag{41}$$

$$\sum_{j \in N} x_{jit} + \sum_{j \in N} x_{ijt} = 2 z_{it} \qquad \forall i \in N \tag{42}$$

$$z_{0t} \leq m \tag{43}$$

$$w_{it} - w_{jt} \geq q_{it} - \widetilde{M}_{it}(1 - x_{ijt}) \ \forall (i,j) \in A \tag{44}$$

$$0 \leq w_{it} \leq Q z_{it} \qquad \forall i \in N_c. \tag{45}$$

Constraints (40)-(45) are equivalent to constraints (7)-(12).

A lower bound on the original problem can be obtained when the integrality constraints on the variables θ_{rt} are relaxed and the problem is solved to optimality. However, in the experiments of Nananukul (2008) on instances with up to 10 customers, 6 periods and 5 vehicles, this relaxation could only give a lower bound which on average was 0.43% higher compared to the original model PRP1.

4. Exact Solution Algorithms

Exact solution algorithms for PRP are very scant. To the best of our knowledge, three exact algorithms were proposed to solve the PRP: the branch-and-cut with strong reformulation of Ruokokoski et al. (2010) for the PRP with a single uncapacitated vehicle, the branch-and-cut of Archetti

et al. (2011) for the PRP with a single capacitated vehicle, and the branch-and-cut of Adulyasak et al. (2013) for the PRP with multiple capacitated vehicles. Although the branch-and-price algorithm of Bard and Nananukul (2009a) can be used to obtain an optimal solution, the subproblem $SVRP_t$ employs the VRP structure and it is very time consuming to solve the problem to optimality. In their experiments, only instances with 10 customers, 2 periods and 5 vehicles where solved to optimality. Therefore, Bard and Nananukul (2009a, 2010) used a heuristic to handle this subproblem and developed a heuristic based on a branch-and-price framework. We later provide the summary of their heuristic algorithm in Section 5.1.2. We also discuss the Benders decomposition algorithm of Adulyasak et al. (2012a) that was proposed to handle the PRP with demand uncertainty in Section 4.4.

4.1. Branch-and-Cut Algorithm of Ruokokoski et al. (2010)

The problem with an uncapacitated plant and a single uncapacitated truck was considered by Ruokokoski et al. (2010). They used the stronger LSP reformulations, e.g., facility location and shortest path reformulations, to solve the PRP. The formulation is similar to PRP2 but the vehicle index k is dropped and the subtour elimination constraints (29) are replaced with

$$\sum_{i \notin S} \sum_{j \in S} x_{ijt} + \sum_{i \in S} \sum_{j \notin S} x_{ijt} \geq 2 z_{et} \quad \forall S \subseteq N_c : |S| \geq 2, \forall e \in S, \forall t \in T. \quad (46)$$

Ruokokoski et al. (2010) investigated the quality of lower bounds by using different LSP reformulation schemes compared to the basic LSP formulation. The computational results show that the LP relaxation values of the shortest path and facility location reformulations when the subtour elimination constraints are dropped are much improved compared to the basic formulation. The LP bound obtained by the shortest path reformulation is greater than or equal to the LP bound of the facility location reformulation which follows from the proof in Solyalı and Süral (2012), but the difference between the bounds is very small. The facility location reformulation provides better computational performance when adding the valid inequalities for the routing problem (including the subtour elimination constraints) and solving the integer problem using a branch-and-cut process.

In their branch-and-cut implementation, the authors used a heuristic and an exact separation procedure based on a minimum $s-t$ cut problem to detect subtours. They also adapted the generalized comb and 2-matching

inequalities presented by Fischetti et al. (1997) which were developed for a generalized travelling salesman problem (GTSP) to the PRP. The results show that when all valid inequalities are used, the facility location LSP reformulation provides the best LP relaxation. This algorithm can solve problems with 80 customers and 8 periods within about 30 minutes.

4.2. Branch-and-Cut Algorithm of Archetti et al. (2011)

Archetti et al. (2011) studied the PRP with uncapacitated production and a single capacitated vehicle. They used the formulation PRP2 but with the SECs (29) being replaced with (33) and the vehicle index k is dropped. The authors also added the inequalities of Archetti et al. (2007) for the inventory routing problem to strenghthen the inventory and routing parts including the following inequalities:

$$I_{i,t-s-1} \geq \sum_{j=0}^{s} d_{i,t-j} \left(1 - \sum_{j=0}^{s} z_{i,t-j}\right) \quad \forall i \in N_c, \forall t \in T, s = 0, 1..., t-1. \quad (47)$$

The inequalities (47) can be interpreted as follows: if there is no shipment delivered during the time interval $[t-s,t]$, the inventory level in period $t-s-1$ must be sufficient to satisfy the demand in this interval. By adding them to the PRP, they could strengthen the lot-sizing part of the customer replenishment and provide better lower bounds. In Archetti et al. (2011), they also proposed inequalities specifically for the PRP with uncapacitated production.

In the branch-and-cut process, the subtour elimination constraints (33) are removed and only the violated cuts are added iteratively during the branching process. The performance of the algorithm is tested on generated test instances with 14 customers and 6 periods. Most of the instances are solved to optimality within a few seconds.

4.3. Branch-and-Cut Algorithms of Adulyasak et al. (2013)

Adulyasak et al. (2013) extended the branch-and-cut approach of Archetti et al. (2011) to the PRP with multiple vehicles. They compared the performance of two formulations. The first one is the formulation without a vehicle index, i.e., the PRP1 with constraints (11) and (12) being replaced by the GFSECs (19). The authors also developed inequalities to strengthen the routing part of the formulation. The second one is the formulation with a vehicle index, i.e., the PRP2 with constraints (11) and (12) being replaced

by (19). This formulation is enhanced with the inequalities to deal with symmetry breaking constraints which disallow alternative solutions that can be created due to the fact that vehicles are identical.

Both formulations are further enhanced with the inequalities of Archetti et al. (2011). For the formulation without a vehicle index, the authors employed the separation algorithms of Lysgaard et al. (2004), which were developed for the VRP, and they also developed a greedy heuristic algorithm for the new cuts. The formulation with a vehicle index is solved by a branch-and-cut algorithm similar to that of Ruokokoski et al. (2010). The experiments showed that the formulation with a vehicle index is superior in finding optimal solutions. Instances with up to 35 customers, 3 periods and 3 vehicles were solved to optimality in two hours. The formulation without a vehicle index, however, could generally produce better lower bounds at the root node and found better lower bounds for the instances that were not solved to optimality in two hours. The authors also tested the performance of the formulation with a vehicle index with an eight-core machine using parallel computing and instances with up to 50 customers, 3 periods and 3 vehicles were solved to optimality in 12 hours. The approaches of Adulyasak et al. (2013) were also adapted for the PRP with the OU policy and IRP with the ML and OU policies.

4.4. Benders-based Branch-and-Cut for the Stochastic Production Routing Problem of Adulyasak et al. (2012a)

Adulyasak et al. (2012a) specifically addressed the PRP with demand uncertainty and introduced the stochastic PRP (SPRP) under demand uncertainty in a two-stage decision process. The first stage consists of making setup and routing decisions before the realization of demand, and the second stage involves production and delivery quantity decisions made when the demand becomes known. They developed exact solution approaches based on Benders decomposition to solve the problem and two different Benders reformulation schemes were proposed. The first Benders reformulation separates the first- and second-stage decisions in the master and subproblems, respectively. In the second reformulation, the routing decisions are also projected out to the subproblems. The two Benders reformulations were enhanced with inequalities to improve the lower bound, aggregate Benders cuts using scenario groups, and Pareto-optimal cuts (Magnanti and Wong, 1981). The Benders algorithm was implemented within a branch-and-cut framework in which Benders cuts are generated at the nodes of the branching tree for the

master problem. The computational experiments show that this implementation outperforms the standard implementation of the Benders algorithm where the master problem is solved from scratch at each iteration. This new algorithm also provides superior results to the branch-and-cut approach of Adulyasak et al. (2013) when solving a large number of scenarios. They further discussed the reoptimization capabilities of the Benders approach which can be particularly useful in two stochastic environments, namely, a sample average approximation scheme (SAA) to handle a large number of scenarios, and a rolling horizon framework (RH) for a dynamic and stochastic variant of the PRP.

5. Heuristics

This section provides a comprehensive review of the heuristics for the PRP. We categorize the heuristics into two groups, i.e., general heuristics and metaheuristics. The details on publicly available benchmarks and recent computational results of the heuristics are presented in Section 5.3.

5.1. General Heuristics

This section presents a review of general heuristics for the PRP, i.e., decomposition-based approach, branch-and-price heuristic, MIP heuristic and iterative MIP heuristic.

5.1.1. Decomposition-Based Heuristics

These heuristics solve the PRP by decomposing the problem into production and distribution planning subproblems. The initial solution is obtained by sequentially solving each problem and a heuristic procedure is called to improve the solutions. This approach was first introduced by Chandra (1993) and Chandra and Fisher (1994) to solve the multi-product PRP. The integrated problem is decoupled into the capacitated lot-sizing problem and the distribution scheduling problem. The lot-sizing problem is solved to optimality and a distribution schedule for each period is produced by applying a simple heuristic together with a 3-opt procedure (Lin, 1965). The result is further improved by allowing production shifting across periods if the total cost is reduced. This heuristic algorithm provides approximately 6% cost savings compared to the uncoordinated approach with no improvement heuristic procedure on the small test instances.

Instead of focusing only on the lot-sizing part in the first phase of the algorithm, Lei et al. (2006) incorporated the distribution part and developed a two-phase heuristic to solve the PRP with multiple plants and a heterogeneous fleet of vehicles. In the first phase, they assume that the deliveries are made by direct shipments from plants to customers and solve the integrated lot-sizing problem with direct shipment. In the second phase, the decisions in the first phase except the direct transportation routes are fixed and the authors used a VRP heuristic to determine the routes for each vehicle at each plant in each period. In the experiments using instances with one plant and up to 12 customers, 2 vehicles and 4 periods, this approach could generally provide better solutions with much shorter computing times compared to the solutions obtained by solving the full model with CPLEX. They also tested the algorithm using a real world dataset.

Boudia et al. (2008) developed an improved decomposition based approach by first determining production lot sizes as large as possible to cover some future periods. The distribution plan in each period is constructed by the savings algorithm (Clarke and Wright, 1964). The algorithm finds opportunities to reduce production costs by adopting the Wagner-Whitin algorithm (Wagner and Whitin, 1958) for the LSP. Then, a local search procedure based on 3-opt moves, insertion, and swap heuristics is called to improve the solution. The algorithm is called H2 and it is tested on the large instances generated by Boudia et al. (2005) with 50-200 customers and 20 periods. It provides 10%-15% cost savings compared to the two-phase decoupled heuristic, called H1, which basically provides a solution from the production plan identified by the Wagner-Whitin method, and the delivery plans generated by a 3-opt procedure.

5.1.2. Branch-and-Price Heuristic

Bard and Nananukul (2009a, 2010) presented a heuristic based on the branch-and-price framework using the RMP and subproblems as described in Section 3.2. The branch-and-price scheme is a decomposition based procedure which involves a branching process. At each branching node, column generation is performed to add variables to the RMP and this updated RMP is solved again until an optimal LP relaxation value of the node is found. Then, the branching process continues until an optimal solution to the original problem is obtained. The readers are referred to Barnhart et al. (1998) for more details on branch-and-price.

The branching process starts with the production setup variables (y_t)

until all these variables have integer values. Subsequently, the variables θ_{rt} are considered. Branching on θ_{rt} directly, however, results in an unbalanced branching tree. When the variable θ_{rt} is set to one, the delivery plan r is used and all corresponding q_{it} and x_{ijt} variables are fixed. But when $\theta_{rt} = 0$, it is very difficult to manage the variables to only exclude the delivery plan r. Therefore, it is more appropriate to branch on the x_{ijt} variables. This branching scheme is similar to the branch-on-edge approach presented in Bramel and Simchi-Levi (2001). The depth-first-search strategy is used to quickly find an incumbent solution. To improve the branch-and-price procedure, several features are included in the process. First, an initial solution is generated by the tabu search heuristic presented by Bard and Nananukul (2009b). Second, during the column generation process, instead of solving the subproblems to optimality, they are solved by the separation based heuristic algorithms of Bard and Nananukul (2009a). Third, the branching scheme is modified to branch on groups of variables. And fourth, a rounding heuristic procedure is used. With these modifications, the performance of the branch-and-price process is substantially improved. The experiments on instances with up to 50 customers and 8 periods showed that this branch-and-price heuristic provides better solutions than those obtained by CPLEX (solution costs are improved by 12.2% on average) within one hour of computing time.

5.1.3. MIP-Based Heuristic

Archetti et al. (2011) decomposed the PRP into the uncapacitated lot-sizing and inventory-routing subproblems and developed a heuristic to solve the decomposed problems. The algorithm starts by fixing production quantities equal to the demand in each period and solving the IRP by a heuristic procedure. In this process, each retailer is selected sequentially and a search tree is explored to determine the time periods and vehicles used to serve that retailer. After that, the uncapacitated lot-sizing subproblem is solved to further explore whether the production plan can be improved by shifting some production quantity to reduce the production and inventory costs. A heuristic procedure is applied to the current solution to obtain further improvements by removing two retailers, and then a problem is formulated to find the minimum insertion cost of these retailers. If the total cost is reduced, the uncapacitated LSP subproblem is solved again and the process is repeated until there is no improvement. The authors evaluated the performance of this algorithm by comparing it to the best solutions found by the exact branch-and-cut solution procedure as described in Section 4.2 on in-

stances with 14 customers, 6 periods and one vehicle. This heuristic provides solutions within 1% of optimality in a few seconds.

5.1.4. Iterative MIP Heuristic

Absi et al. (2013) introduced an iterative MIP heuristic to solve the PRP with uncapacitated production. The MIP is formulated by replacing the routing costs and variables in the original PRP model with fixed costs SC_{vit} representing the approximate cost of visting customer i in period t with vehicle v. At each iteration, this MIP is solved to optimality (or until a maximum CPU time is reached) to obtain production, inventory and customer visit decisions. Then, a heuristic is called to determine the routes to serve the visited customers. Next, the costs SC_{vit} are set to a minimum insertion cost corresponding to the current solution and the next iteration is performed. A diversification mechanism that modifies visit costs is also applied when the current solution is not improved for a certain number of iterations. The process stops when a maximum number of iterations is reached. The authors proposed two variants of the iterative approach. In the first variant, an individual vehicle capacity is taken into account when solving the MIP and each route is determined by a TSP heuristic, while an aggregate vehicle capacity is considered and the routes are determined by a VRP heuristic in the second variant. The results show that the second variant is superior to the first and can generally produce better solutions in a shorter computing time. Although the performance of this algorithm relies heavily on the performance of the MIP solver, the results show that it outperformed other heuristics on the data set with uncapacitated production of Archetti et al. (2011).

5.2. Metaheuristics

We first describe the different metaheuristics in Sections 5.2.1-5.2.4 and then provide a summary of the computational results of these metaheuristics in Section 5.3.

5.2.1. Greedy Randomized Adaptive Search Procedure (GRASP)

GRASP was first introduced by Feo and Resende (1995). Basically, the procedure consists of two main phases, i.e., construction and local search. In the construction phase, an initial solution is provided via an adaptive randomized greedy algorithm. Then the local search phase is applied to improve the solution.

A GRASP for the PRP was developed by Boudia et al. (2007). In the construction phase, an initial solution is generated by sequentially developing a production and delivery plan. Starting from the first period onwards, the production plan is preliminarily determined by producing a sufficient amount to cover the demand in the period without excess production which creates inventory at the plant. Then, delivery routes in the period are constructed by an insertion algorithm. Next, the algorithm checks if some customer demands can be moved to prior periods without violating production, inventory and vehicle capacities and the insertion process is again performed to insert these quantities. Then, the saving heuristic is called to find a better routing solution. All routing plans are fixed and the production plan improvement algorithm is applied to shift production quantities to combine with a production plan in earlier periods if the cost is lower, i.e., the incurred storage cost at the plant is less than the setup cost. In the local search phase, the routing plan of each period is improved by using a 3-opt procedure, inserting, and swapping. Moves across periods are also considered if the cost can be reduced. Boudia et al. (2007) also developed a path relinking procedure (see Glover (1996)) as a post-processor. In this process, solutions obtained during the GRASP are ranked according to their total cost and a limited number of solutions are stored in a pool of elite solutions. Then, any two solutions in the pool are chosen to create a new solution by transferring some delivery quantities in one of these solutions to another period according to the delivery quantities in the other solution to reduce the differences between these two solutions. This process could slightly improve the solutions obtained by GRASP.

5.2.2. Memetic Algorithm (MA)

Informally speaking, a memetic algorithm is a modified genetic algorithm (GA) that uses some form of local search to improve solutions. The basic idea of a genetic algorithm is to generate new solutions from a population of initial solutions which are represented by chromosomes (or bitstrings) using natural evolution, i.e., crossover or mutation, to create new offsprings. In a memetic algorithm, a local search procedure is additionally applied to improve both the initial population and offsprings of the genetic algorithm. This approach was first introduced by Moscato (1999). Boudia and Prins (2009) applied this approach to the PRP with a special feature, called memetic algorithm with population management (MA|PM) (Sörensen and Sevaux, 2006).

In the study of Boudia and Prins (2009), an initial population is created

through a simple heuristic procedure that preliminarily sets a production plan in each period equal to the total demand. Then, a savings heuristic is used to generate the delivery plan and the production plan is adjusted by the Wagner-Whitin algorithm. Next, the algorithm uses a crossover to generate new offsprings. In the following step, the authors adopted the local search of Boudia et al. (2007) to improve the generated offsprings and applied the population management approach to select them, i.e., the new offsprings are accepted only if they improve the current solution by more than a threshold value. The algorithm terminates when the maximum number of iterations is reached.

5.2.3. Tabu Search

The concept of tabu search was introduced by Glover (1989). In this procedure, the search moves at each iteration from the current solution to the best neighbor solution. In order to avoid cycling and to get out of local optima, all visited solutions are stored in a tabu list and these solutions are forbidden from the search procedure. The tabu search approach is known to be one of the most efficient solution methods for the VRP (Gendreau et al., 2001).

A tabu search for the PRP was proposed by Bard and Nananukul (2009b). They used a reactive tabu list of variable size and this method is called reactive tabu search (RTS) (Battiti and Tecchiolli, 1994). In their algorithm, an initial solution is created by solving a modified PRP obtained by dropping the routing constraints (8)-(12), removing variables x_{ijt}, and assuming that the delivery cost is equal to the round trip transportation cost. Then, a subsequent routing decision is made by applying a capacitated vehicle routing problem (CVRP) subroutine based on a tabu search proposed by Carlton and Barnes (1996). The generated solution is improved by a local search procedure using swap or transfer moves. The swap move examines two customers in two periods and exchanges maximum possible delivery quantities between these two customers. The transfer move finds the delivery quantity of a customer that can be combined with another delivery in a previous period in order to reduce the transportation cost. The moves that lead to an improved solution are stored in the tabu list and infeasible solutions are not allowed.

Armentano et al. (2011) developed a tabu search with path relinking (TSPR) procedure for the PRP. In their algorithm, an initial solution is created by setting delivery quantities equal to demands and applying the

Wagner and Whitin (1958) algorithm and the Clarke and Wright (1964) savings algorithm to obtain the production and routing decisions, respectively. In the neighborhood search of Armentano et al. (2011), a move similar to the transfer move of Bard and Nananukul (2009b) is used but it also allows a delivery quantity to be combined with another delivery in a future period. At each iteration, the algorithm also solves an LP to optimize the production and inventory quantities at the plant. The local search terminates when it reaches the maximum number of iterations. Then, a path relinking procedure similar to that of Boudia et al. (2007) is used to diversify the search.

5.2.4. Adaptive Large Neighborhood Search (ALNS)

The adaptive large neighborhood search (ALNS) framework was introduced by Ropke and Pisinger (2006) to solve the VRP with pickup and delivery. The basic idea of the ALNS is to repeatedly destroy and repair a part of a solution to obtain an improved solution using search operators. These operators are probabilistically selected based on empirical scores. This procedure was recently applied to various routing applications.

The ALNS of Adulyasak et al. (2012b) incorporated three main features. First, an enumeration scheme is used to create several different initial solutions. Each initial solution is generated by solving the two decomposed problems, similarly to the approach of Bard and Nananukul (2009b), and local branching inequalities (Fischetti and Lodi, 2003) are applied to generate another solution with a different production configuration. Second, the authors developed two types of operators, called selection and transformation. At each iteration, one operator of each type is probabilistically selected. The selection operator is applied first to create a list of node candidates (customer-period combinations) and then the transformation operator is applied to remove and reinsert node candidates in the list to the current solution. Third, when a new solution is found during the transformation process, a minimum cost flow (MCF) problem is solved to optimize the production, inventory and delivery quantities. The algorithm stops when it reaches the maximum number of iterations. Since the quantity flow part is optimized by the MCF, this algorithm is referred to as an optimization-based ALNS (Op-ALNS).

5.3. Benchmark Instances and Computational Evaluation of the Heuristics

In this section, we provide details on the benchmark instances that are publicly available as well as recent computational results. The first bench-

mark was introduced by Boudia et al. (2005) and it consists of three problem sets with 50, 100, 200 customers and 20 time periods; there are 30 instances per set. This dataset was used in the experiments of Boudia et al. (2008) and all the metaheuristics described in the previous section. This dataset can be found on the website: `https://sites.google.com/site/YossiriAdulyasak/publications`. The second benchmark was introduced by Archetti et al. (2011). It consists of sets of problems with 14, 50, 100 customers and 6 time periods. These instances are smaller compared to those of Boudia et al. (2005). However, the set contains instances with four different aspects, i.e., standard, high production cost, high transportation cost and zero customer inventory cost. This data set was used in the experiments of Archetti et al. (2011), Absi et al. (2013) and Adulyasak et al. (2012b) and it can be downloaded at: http://www-c.eco.unibs.it/~bertazzi/ml.zip.

A comparison of the computational performance of the heuristics of Boudia et al. (2008) and the metaheuristics on the Boudia et al. (2005) dataset is reported in Tables 2 and 3. The computational experiments were conducted with the test instances from Boudia et al. (2005). All test evaluations were performed on workstations with comparable CPU performances. The decomposition-based heuristics H1 and H2 were very quick but the quality of the solutions is not comparable to the metaheuristics. The best solutions are provided by the ALNS by Adulyasak et al. (2012b). Although this algorithm spent higher computing times on average compared to other metaheuristics, better solutions were obtained in the early stage of the algorithm as reported in Adulyasak et al. (2012b), where the average computing times are comparable to those of the GRASP developed by Boudia et al. (2007).

Table 2: Average total costs obtained by different heuristics on the Boudia et al. (2005) benchmark

Set	$\lvert N_c \rvert$	l	H1[1]	H2[2]	GRASP[3]	MA|PM[4]	RTS[5]	TSPR[6]	ALNS[7]
B1	50	20	511579	453462	443264	393263	369662	361704	**346878**
B2	100	20	963649	831019	791839	714627	712294	685898	**636962**
B3	200	20	1312612	1112392	1070026	1001634	1034923	951638	**876761**

[1,2]Boudia et al. (2005)
[3]Boudia et al. (2007)
[4]Boudia and Prins (2009)
[5]Bard and Nananukul (2009b)
[6]Armentano et al. (2011)
[7]Adulyasak et al. (2012b)

Table 4 shows the results obtained by the heuristic $\mathcal{H}$ of Archetti et al.

Table 3: Average computing times in seconds obtained by different heuristics on the Boudia et al. (2005) benchmark

Set	$\lvert N_c \rvert$	l	H1[1]	H2[2]	GRASP[3]	MA|PM[4]	RTS[5]	TSPR[6]	ALNS[7]
B1	50	20	0.1	0.2	93.5	172.7	330.6	317.0	481.3
B2	100	20	0.5	1.1	415.9	1108.1	975.6	1147.6	1569.9
B3	200	20	2.1	10.4	1893.8	4098.5	2492.3	3926.4	5794.2

[1,2,3,4]executed on 2.30 GHz PC
[5]executed on 2.53 GHz PC
[6]executed on 2.80 GHz PC
[7]executed on 2.10 GHz Duo CPU PC

(2011), the ALNS of Adulyasak et al. (2012b) and the best variant of the iterative MIP heuristic (IM) of Absi et al. (2013) on the Archetti et al. (2011) benchmark test set. For the 467 instances of A1 solved to optimality, the first variant of IM provides an average gap of 1.22%, whereas $\mathcal{H}$ and ALNS have a gap of 1.71% and 2.66% respectively. For the multi-vehicle instances A2 and A3, the best version of the IM heuristics is the second variant. We note that for the uncapacitated production problems, the IM heuristic of Absi et al. (2013) generally provides the best solutions.

Table 4: Average total costs and computing times in seconds obtained by different heuristics on the Archetti et al. (2011) benchmark

Set	$\lvert N_c \rvert$	l	Average total cost			Average CPU time		
			$\mathcal{H}$[1]	ALNS[2]	IM[3]	$\mathcal{H}$[1]	ALNS[2]	IM[3]
A1	14	6	180830	181803	n/a	-	9.0	11.3
A2	50	6	592608	590210	**587973**	11.0	46.6	24.4
A3	100	6	1092509	1089589	**1084320**	188.4	212.5	80.8

- computational times are negligible
[1] executed on 2.40 GHz PC
[2] executed on 2.10 GHz CPU PC
[3] executed on 2.67 GHz CPU PC

6. Future Research Directions

As an integration of several areas in production and distribution planning, there are many future research directions that have not been explored. We summarize them here.

Exact Algorithms. Although some exact algorithms were proposed, the best ones are still limited to medium instance size with a relatively small number of time periods. Several techniques that were used to enhance these algorithms were adapted from the methods developed for the LSP and VRP. However, inequalities or formulations that strengthen the coordination between these two parts of the problem can be further explored. Moreover, a complex decomposition based approach like branch-cut-and-price may be effective to handle the PRP. By applying this approach to the formulations where subtour elimination constraints are used, a column generation technique can be used to control the flow part of the problem where the subtour elimination constraints are handled by a cutting-plane technique.

Other variants of the LSP. As a major component of the PRP, one may consider other interesting variants of the LSP. For example, the multi-product problem where the production setup for each product must be done separately. Although some heuristics have been proposed for this variant (Fumero and Vercellis, 1999 and Armentano et al., 2011), no studies have discussed exact methods to solve the problem. One can also consider the multi-product variant where production startups must also be taken into account. We refer to Jans and Degraeve (2008) for the details of other variants of the LSP.

PRP with customer visit and delivery time windows. Time windows are one of the most common issues in transportation operations. There are two different types of time windows that can be considered in the PRP. The first type, called customer visit time windows, is imposed on periods where some customers should be visited to satisfy operational requirements. The second type, called delivery time window, is imposed during the day of delivery and is a well-known variant of the VRP.

Robust PRP. In stochastic environments, it is possible that a probabilistic description of the uncertainty is not available and one cannot use the SPRP to solve the problem. In this case, instead of minimizing the expected total cost, one is interested in obtaining a solution that is immune to any realization of the uncertainty in a given set and therefore the product availability must be guaranteed. A study of the robust approach for the IRP can be found in Solyalı et al. (2012).

Tactical PRP. In many applications, customers must be clustered and a driver is assigned to serve a specific cluster. The cluster decisions are fixed for a long term horizon. Examples of this variant can be found in Michel and Vanderbeck (2012) and Coelho et al. (2012a). One possible approach is to decompose the problem. The first problem would be used to identify clusters, while the other problem would be a modified PRP to determine the remaining decisions.

7. Conclusion

This paper provides an in-depth review of the production routing problem to support the growing interest in this research area. We first described the relevance of this problem and of the other two integrated supply chain planning problems, namely, the lot-sizing problem with direct shipment and the inventory routing problem. As a combination of the lot-sizing and vehicle routing problem, the production routing problem has an interesting, but complex, structure. Therefore, the majority of the studies proposed decomposition-based heuristics and metaheuristics to handle the problem, while few exact algorithms were developed. We discussed these techniques that have been developed for several variants of the PRP and summarized the computational results. Although various contributions have been made to this problem, there are still a number of interesting issues that have not been addressed. We encourage researchers in this field to pursue further developments in this promising research area.

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